Exam 3 Review

This Exam will be on April 17th at the beginning of class time, 10:30 am until 11:45 am. You are allowed a non-graphing calculator without CAS (computer algebra system). You are also allowed both sides of a standard sized printer paper (like all the worksheets) as an equation sheet. This sheet must be handwritten (with pen or pencil). It cannot be typeset or have printed equations. Your equation sheet will be turned in with the exam, so it must have your name in the top left corner.

1. Evaluate the following:

   (a) $\iint_R x^2 + ydA$, where R is the region between $y = x^2 - 1$ and $y = x$
   (b) $\iint_R xy^2 dA$ where R is the region between $x = y^2$ and $x = 2 - 2y^2$

2. Set up the following integrals for finding the surface area of a region:

   (a) Σ: the region on the graph of $g(x, y) = x^2 + y^2$ that lies above the triangle with vertices $(1, 1), (-1, 1), (2, 0)$ on the $x - y$ plane
   (b) Σ: the region on the graph of $g(x, y) = x + y$ that lies above the disk with radius 2 centered on the origin.
   (c) Σ: the region on the graph of $g(x, y) = x^2 + 1$ that lies above the annulus with inner radius 1 and outer radius 2 centered on the origin.

3. Evaluate the following integrals:

   (a) $\iint_R 2x^2 + y^2 dA$, R is the pie wedge given by $0 \leq \theta \leq \pi/3$ and $0 \leq r \leq 2$
   (b) $\iint_R \frac{1}{x^2+y^2+1} dA$, where R is the annulus centered on the origin with inner radius 2 and outer radius 4.
   (c) $\iint_R \sqrt{x^2 + y^2} dA$ where R is the unit disk.

4. Evaluate the following integrals:

   (a) $\iiint_D x^2 + y^2 + z^2 dV$ where D is the cylindrical shell with inner radius 1, outer radius 2, which is bounded by $z = 1$ and $z = 4$
   (b) $\iiint_D x + ydV$ where D is the region represented by $[0, 1] \times [\pi/3, \pi/4] \times [-1, 1]$ in $r - \theta - z$ space.

5. Evaluate the following integrals:

   (a) $\iiint_D x^2 + y^2 dV$, where D is the volume given by the inequalities $x^2 + y^2 + z^2 \leq 1$ and $z \geq 0$
   (b) $\iiint_D x - y^2 dV$, where D is the spherical shell with inner radius 1 and outer radius 2
   (c) $\iiint_D e^{x+y+z} dV$ where D is the cone given by $\varphi \leq \pi/4$
   (d) $\iiint_D \frac{r}{x} dV$ where D is the lime wedge given by $-\pi/3 \leq \theta \leq \pi/3$

6. Set up the following integrals:

   (a) Consider the sphere of radius 3 and let Σ be the cap made by $\varphi \leq \pi/3$. Set up the integral which gives you the surface area of Σ
   (b) Consider the earth modeled as a sphere of radius 3963 miles, and let Σ be the tropics, which are defined to be all the points on the surface of the earth between 23.5 degrees above the equator and 23.5 degrees below the equator. Set up the integral that would give you however many square miles are in the tropics (of both sea and land). (Note: you will need to convert everything to radians for the calculus to work out)

7. Let $S(u, v) = \langle u, u^2 + v \rangle$, and let $R_{uv} = [0, 1] \times [0, 2]$. Plot $R = S(R_{uv})$ on the $x - y$ plane. What is the area of R?

8. For each of the following, what are the dimensions of the described objects and what are the dimensions of the spaces these objects are contained within. Write down the notation for the corresponding integral which gives you the arc-length/surface area/volume of the region. (You don't have to evaluate, you just need to write the correct notation)

   (a) The region on the graph of $g(x, y) = \sin(xy)$ which lies above $[0, 1] \times [1, 2]$ on the $x - y$ plane
   (b) The inequality $x^2 + y^2 - z^2 \geq 1$

(c) the equaltity $2x^2 + y^2 + z^2 = 1$

(d) the set of solutions to the system of equations: $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 = z$

(e) the region parameterized by $S(s,t) = \langle s, s+t, s^2+t^2 \rangle$ for $0 \le s \le 1$ and $1 \le t \le 3$

(f) the region parameterized by $r(t) = \langle t, t^2, t^3, t^4 \rangle$

9. Calculate the following:

   (a) $\int_{\partial R} x^2 - y^2 ds$ where R is the region given by $x^2 + y^2 \le 1$

   (b) $\int_{\partial R} x^2 + y ds$ where R is the triangle with vertices $(1,1)$, $(1,-1)$, $(0,-1)$

   (c) $\int_{\partial R} (x-y)^2 ds$ where R is the square with vertices $(0,0)$, $(1,1)$, $(0,2)$, $(-1,1)$

10. For each vector field V, plot V (with at least 9 vectors), check if V is conservative, and give the corresponding potential function if applicable.

    (a) $V(x,y) = \langle x^2 + y, x - y^2 \rangle$

    (b) $V(x,y) = \langle x, y \rangle$

    (c) $V(x,y) = \langle y, x \rangle$

    (d) $V(x,y) = \langle y, -x \rangle$

11. For each of the following vector fields, find the curl and divergence:

    (a) $V(x,y,z) = \langle xy, yz, zx \rangle$

    (b) $V(x,y,z) = \langle x+y, y+z, z+x \rangle$

    (c) $V(x,y,z) = \langle (x+z)^2, (y+x)^2, (z+y)^2 \rangle$

12. Things to Know:

    (a) How to parameterize spheres, cylinders, circles, lines, line segments (being mindful of orientation), elliipses

    (b) what inequalities look like in cylindrical coordinates

    (c) What inequalities look like in spherical coordinates

    (d) trig integrals

    (e) how to push points, curves, and regions through parameterizations