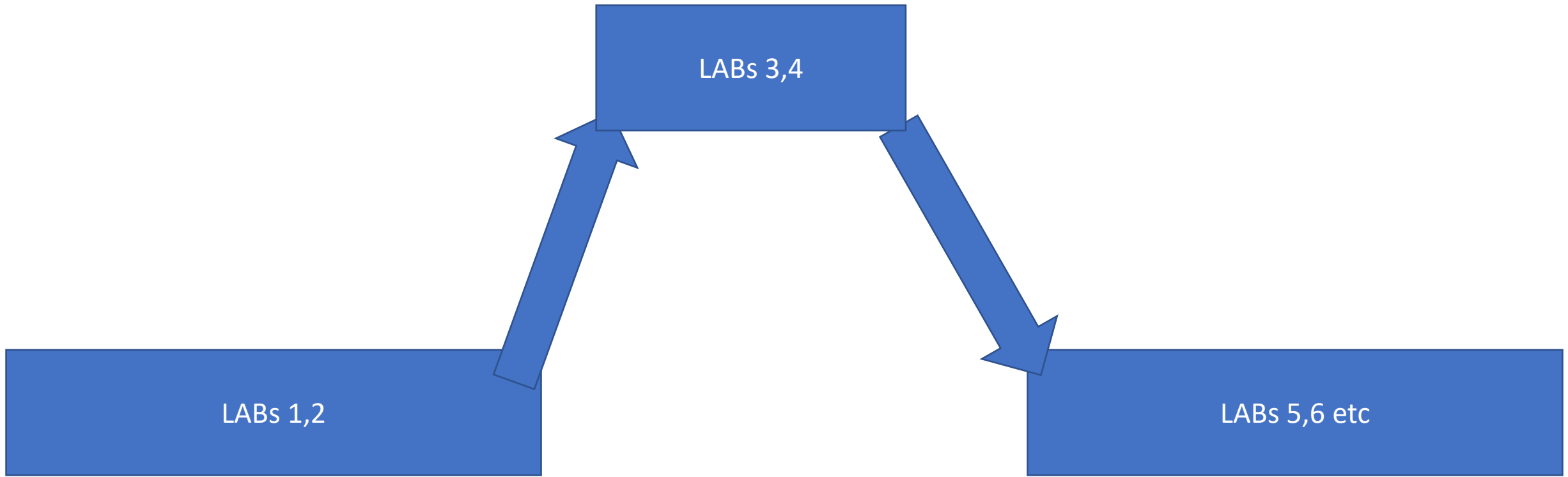


Chapter 3

Dr Wayne Stewart

MS sixth edition





ASSIGNMENT 1: Hints

11: MS 2.72

This is going to be one wild ride. Let's start by reading in our data and separating it into old and new location readings.

```
voltage.df <- v$VOLTAGE
#voltage.df = read.csv("data/VOLTAGE.csv");
#names(voltage.df)
voltage.old.df = voltage.df[voltage.df$LOCATION == "OLD",]
voltage.new.df = voltage.df[voltage.df$LOCATION == "NEW",]
```

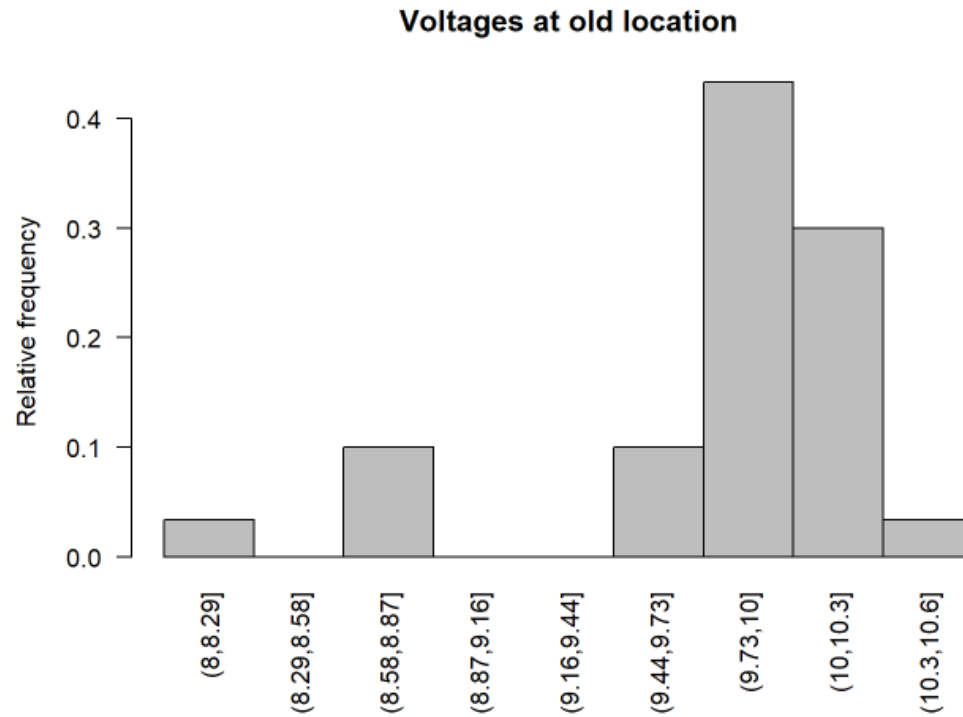
a

I'll first summarize the data/histogram creation process using the provided table.

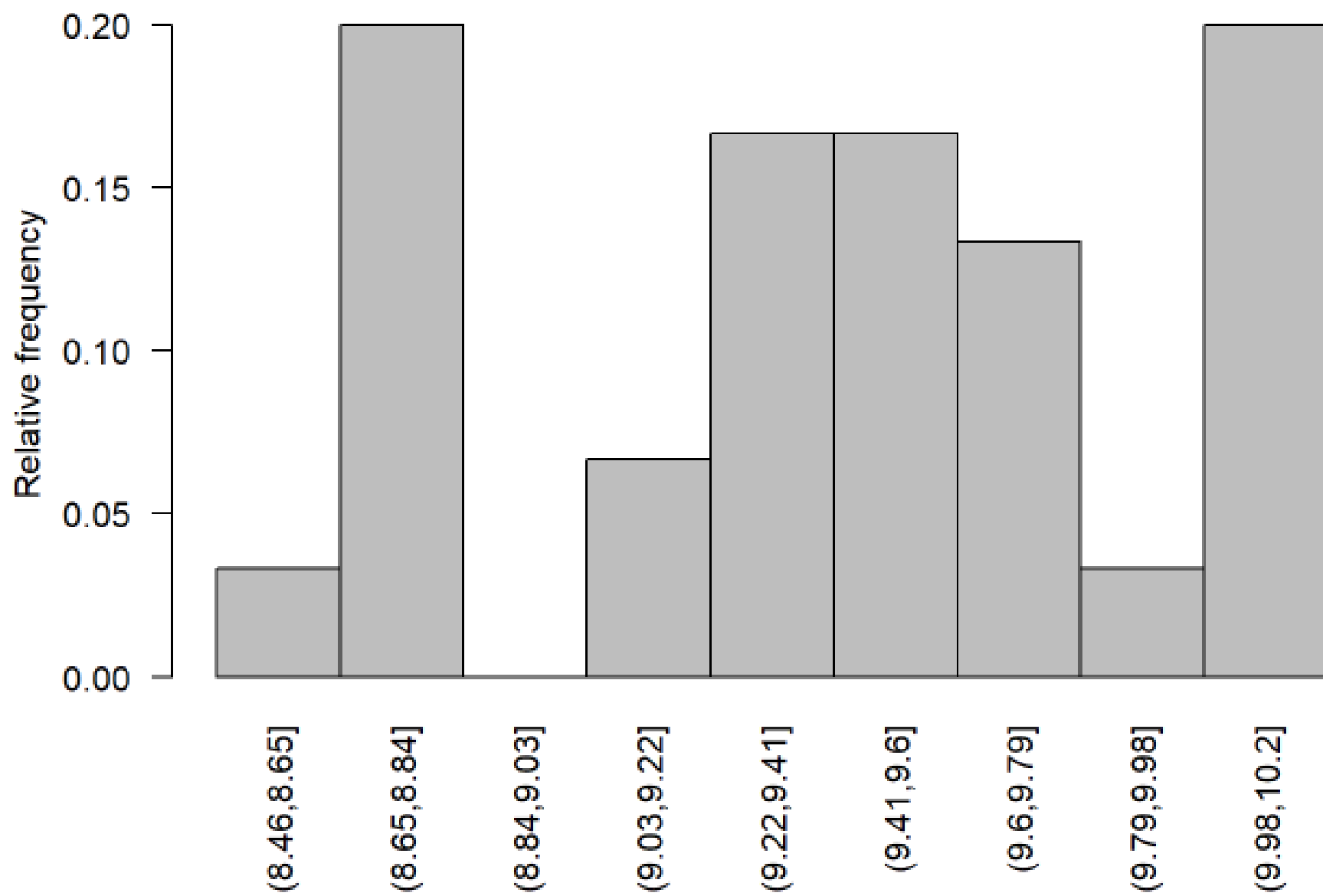
Class	Interval	Data	Frequency	Relative frequency
1	(8.0000,8.2900]	8.05	1	0.0333
2	(8.2900,8.5800]	-	0	0.0000
3	(8.5800,8.8700]	8.72, 8.72, 8.80	3	0.1000
4	(8.8700,9.1600]	-	0	0.0000
5	(9.1600,9.4500]	-	0	0.0000
6	(9.4500,9.7400]	9.55, 9.70, 9.73	3	0.1000
7	(9.7400,10.0200]	9.80, 9.80 9.84, 9.84, 9.87, 9.87, 9.95, 9.97, 9.98, 9.98, 10.00, 10.10, 10.02	13	0.4333
8	(10.0200,10.3100]	10.03, 10.05, 10.5, 10.12, 10.15, 10.15, 10.26, 10.26, 10.29	9	0.3000
9	[10.3100,10.6000]	10.55	1	0.0333
Total	-	-	30	1

Function is not asked for but you are free to make an algorithm and create a function if you wish

```
rel.freq.hist = function(v, bins = 9, main = "Relative frequency histogram", ...) {  
  left.stop = min(v) - 0.05;  
  right.stop = max(v) + 0.05;  
  range = right.stop - left.stop;  
  delta = range / bins;  
  s = seq(left.stop, right.stop, by = delta);  
  cuts = cut(v, breaks = s);  
  tab = table(cuts);  
  barplot(tab / sum(tab), space = 0, main = main, las = 2, ylab = "Relative frequency", ...)  
}  
rel.freq.hist(voltage.old.df$VOLTAGE, main = "Voltages at old location");
```



Voltages at new location



Old

```
mode = function(x) {  
  # Remove duplicates  
  uniq = unique(x);  
  # Select most common from list of uniques  
  uniq[which.max(table(match(x, uniq)))]  
}  
mean(voltage.old.df$VOLTAGE)
```

```
## [1] 9.803667
```

```
median(voltage.old.df$VOLTAGE)
```

```
## [1] 9.975
```

```
mode(voltage.old.df$VOLTAGE)
```

```
## [1] 9.98
```

The “expected” voltage in the old location is about 9.80 V, and the voltage separating the high and low readings is about 9.98 V. The most common voltage is 9.98 V.

Probability

OBJECTIVE

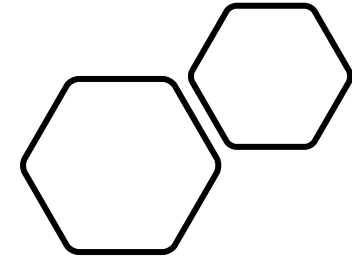
To present an introduction to the theory of probability and to suggest the role that probability will play in statistical inference

CONTENTS

- 3.1 The Role of Probability in Statistics
- 3.2 Events, Sample Spaces, and Probability
- 3.3 Compound Events
- 3.4 Complementary Events
- 3.5 Conditional Probability
- 3.6 Probability Rules for Unions and Intersections
- 3.7 Bayes' Rule (*Optional*)
- 3.8 Some Counting Rules
- 3.9 Probability and Statistics: An Example

- **STATISTICS IN ACTION**


- Assessing Predictors of Software Defects in NASA Spacecraft Instrument Code





Definition 3.1


An **experiment** is the process of obtaining an observation or taking a measurement.





Definition 3.2

A **simple event** is a basic outcome of an experiment; it cannot be decomposed into simpler outcomes.



Example 3.1

Listing Simple Events for a Coin-Tossing Experiment

Solution

Two coins are tossed and the up faces of both coins are recorded. List all the simple events for this experiment.

Even for a seemingly trivial experiment, we must be careful when listing the simple events. At first glance the basic outcomes seem to be Observe two heads, Observe two tails, Observe one head and one tail. However, further reflection reveals that the last of these, Observe one head and one tail, can be decomposed into Head on coin 1, Tail on coin 2 and Tail on coin 1, Head on coin 2.* Thus, the simple events are as follows:

1. Observe HH
2. Observe HT
3. Observe TH
4. Observe TT

(where H in the first position means “Head on coin 1,” H in the second position means “Head on coin 2,” etc.).



Definition 3.3

The **sample space** of an experiment is the collection of all its simple events.




TABLE 3.1 Experiments and Their Sample Spaces

Experiment: Toss a coin and observe the up face.

Sample space: 1. Observe a head
2. Observe a tail

This sample space can be represented in set notation as a set containing two simple events

$$S: \{H, T\}$$

where H represents the simple event Observe a head and T represents the simple event Observe a tail.

Experiment: Toss two coins and observe the up face on each.

Sample space:

1. Observe HH
2. Observe HT
3. Observe TH
4. Observe TT

This sample space can be represented in set notation as a set of four simple events

$S: \{HH, HT, TH, TT\}$

Make sure you are reading the book

Definition 3.4

The **probability** of a event (simple or otherwise) is a number that measures the likelihood that the event will occur when the experiment is performed. The probability can be approximated by the proportion of times that the event is observed when the experiment is repeated a very large number of times.* For a simple event E , we denote the probability of E as $P(E)$.

Rules for Assigning Probabilities to Simple Events

Let E_1, E_2, \dots, E_k be the simple events in a sample space.

1. All simple event probabilities *must* lie between 0 and 1:

$$0 \leq P(E_i) \leq 1 \quad \text{for } i = 1, 2, \dots, k$$

2. The sum of the probabilities of all the simple events within a sample space must be equal to 1:

$$\sum_{i=1}^k P(E_i) = 1$$

Definition 3.5

An event is a specific collection of sample points (or simple events).

The Probability of an Event

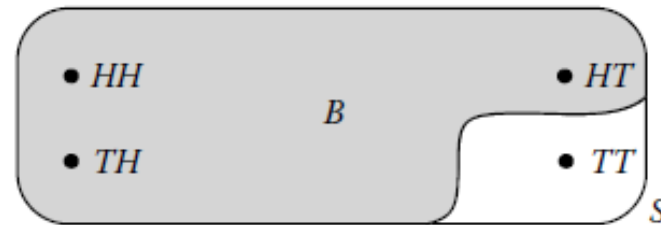
The **probability of an event A** is equal to the sum of the probabilities of the sample points in event A .

FIGURE 3.3

Coin-tossing experiment showing events A and B as collections of simple events



a. Event A



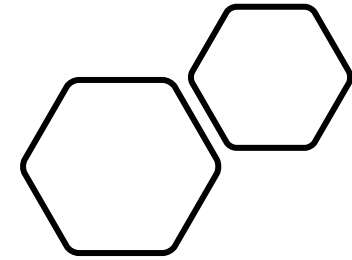
b. Event B

Define the following events:

A : {Observe exactly one head}

B : {Observe at least one head}

Calculate the probability of A and the probability of B .



3.3 Compound Events

An event can often be viewed as a composition of two or more other events. Such events are called **compound events**; they can be formed (composed) in two ways.

Definition 3.6

The **union** of two events A and B is the event that occurs if either A or B , or both, occur on a single performance of the experiment. We will denote the union of events A and B by the symbol $A \cup B$.

$$A \cup B = A \text{ or } B$$

Definition 3.7

The **intersection** of two events A and B is the event that occurs if both A and B occur on a single performance of the experiment. We will write $A \cap B$ for the intersection of events A and B .

$$A \cap B = A \text{ and } B$$

Check out these examples!!

Example 3.5

Finding Probabilities of
Unions and Intersections:
CO Poisoning

The *American Journal of Public Health* (July 1995) published a study on unintentional carbon monoxide (CO) poisoning of Colorado residents. The source of exposure was determined for 1,000 cases of CO poisoning that occurred during a recent six-year period. In addition, each case was classified as fatal or nonfatal. The proportion of the cases occurring in each of 10 source/fatal categories is shown in Table 3.2. Define the following events:

A: {CO poisoning case is caused by fire}

B: {CO poisoning case is fatal}

- Describe the simple events for this experiment. Assign probabilities to these simple events.
- Describe $A \cup B$.
- Describe $A \cap B$.
- Calculate $P(A \cup B)$ and $P(A \cap B)$.

Example 3.6

Probabilities of Unions and Intersections: Die-Tossing Experiment

Consider the die-tossing experiment with equally likely simple events $\{1, 2, 3, 4, 5, 6\}$. Define the events A , B , and C as follows:

$$A: \{\text{Toss an even number}\} = \{2, 4, 6\}$$

$$B: \{\text{Toss a number less than or equal to 3}\} = \{1, 2, 3\}$$

$$C: \{\text{Toss a number greater than 1}\} = \{2, 3, 4, 5, 6\}$$

Find

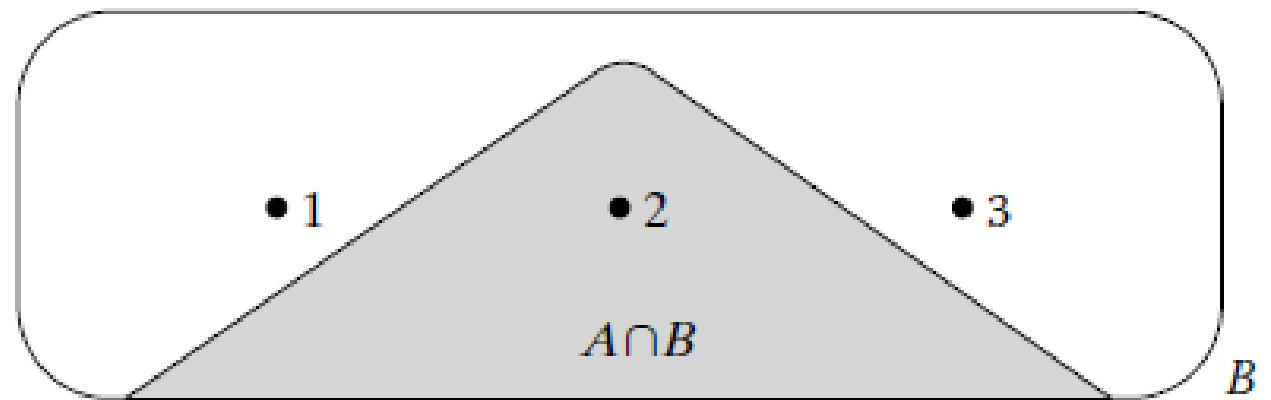
- $P(A \cup B \cup C)$
- $P(A \cap B \cap C)$

Solution

- Event C contains the simple events corresponding to tossing a 2, 3, 4, 5, or 6; event B contains the simple events 1, 2, and 3; and, event A contains the simple events 2, 4, or 6. Therefore, the event that A , B , or C occurs contains all six simple events in S , i.e., those corresponding to tossing a 1, 2, 3, 4, 5, or 6. Consequently, $P(A \cup B \cup C) = P(S) = 1$.
- You can see that you will observe all of the events, A , B , and C , only if you observe a 2. Therefore, the intersection $A \cap B \cap C$ contains the single simple event Toss a 2 and $P(A \cap B \cap C) = P(2) = \frac{1}{6}$.

FIGURE 3.7

Reduced sample space for the die-tossing experiment, given that event B has occurred



Important!!

Formula for Conditional Probability

To find the **conditional probability** that event A occurs given that event B occurs, divide the probability that *both* A and B occur by the probability that B occurs, that is,

$$P(A | B) = \frac{P(A \cap B)}{P(B)} \quad \text{where we assume that } P(B) \neq 0$$

Example 3.10

Conditional Probability Associated with Consumer Complaints

The investigation of consumer product complaints by the Federal Trade Commission (FTC) has generated much interest by manufacturers in the quality of their products. A manufacturer of food processors conducted an analysis of a large number of consumer complaints and found that they fell into the six categories shown in Table 3.3. If a consumer complaint is received, what is the probability that the cause of the complaint was product appearance given that the complaint originated during the guarantee period?

TABLE 3.3 Distribution of Product Complaints

	Reason for Complaint			Totals
	<i>Electrical</i>	<i>Mechanical</i>	<i>Appearance</i>	
During guarantee period	18%	13%	32%	63%
After guarantee period	12%	22%	3%	37%
Totals	30%	35%	35%	100%

Solution

Let A represent the event that the cause of a particular complaint was product appearance, and let B represent the event that the complaint occurred during the guarantee period. Checking Table 3.3, you can see that $(18 + 13 + 32)\% = 63\%$ of the complaints occurred during the guarantee time. Hence, $P(B) = .63$. The percentage of complaints that were caused by appearance and occurred during the guarantee time (the event $A \cap B$) is 32%. Therefore, $P(A \cap B) = .32$.

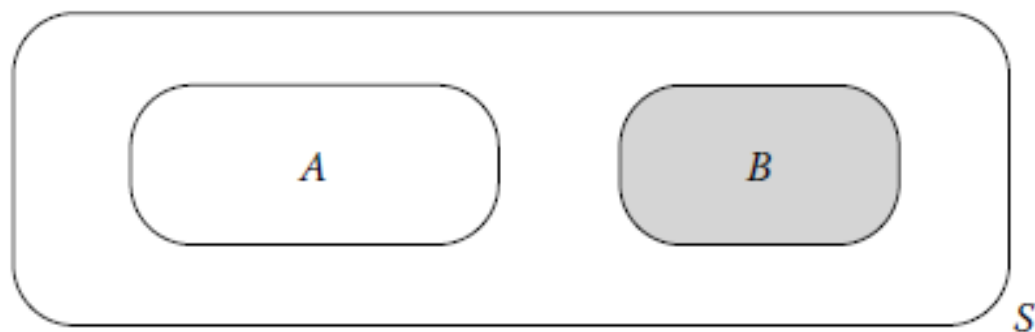
Using these probability values, we can calculate the conditional probability $P(A | B)$ that the cause of a complaint is appearance given that the complaint occurred during the guarantee time:

$$P(A | B) = \frac{P(A \cap B)}{P(B)} = \frac{.32}{.63} = .51$$

Consequently, you can see that slightly more than half the complaints that occurred during the guarantee time were due to scratches, dents, or other imperfections in the surface of the food processors.

FIGURE 3.10

Venn diagram of mutually exclusive events



Definition 3.9

Events A and B are **mutually exclusive** if $A \cap B$ contains no simple events.

Additive Rule for Mutually Exclusive Events

If two events A and B are mutually exclusive, the probability of the union of A and B equals the sum of the probabilities of A and B :

$$P(A \cup B) = P(A) + P(B)$$

Example 3.13

Probability of a Union: Coin Tossing Experiment

Solution

Consider the experiment of tossing two balanced coins. Find the probability of observing at least one head.

Define the events

A : {Observe at least one head}

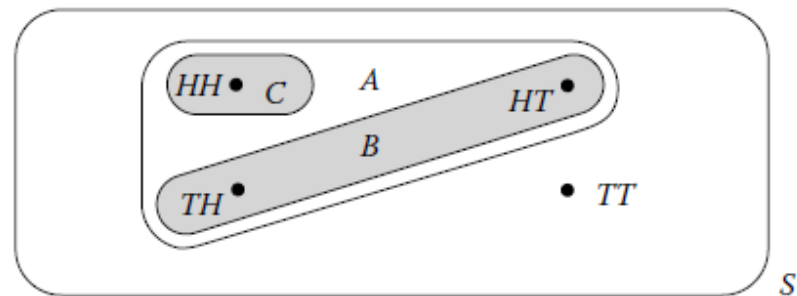
B : {Observe exactly one head}



C : {Observe exactly two heads}

Note that $A = B \cup C$ and that $B \cap C$ contains no simple events (see Figure 3.11). Thus, B and C are mutually exclusive, so that

$$\begin{aligned} P(A) &= P(B \cup C) = P(B) + P(C) \\ &= \frac{1}{2} + \frac{1}{4} = \frac{3}{4} \end{aligned}$$

FIGURE 3.11
Venn diagram for coin-toss
experiment





Multiplicative Rule for Independent Events

If events A and B are independent, the probability of the intersection of A and B equals the product of the probabilities of A and B , i.e.,

$$P(A \cap B) = P(A)P(B)$$


Bayes' Rule

Given k mutually exclusive and exhaustive states of nature (events), A_1, A_2, \dots, A_k , and an observed event E , then $P(A_i | E)$, for $i = 1, 2, \dots, k$, is

$$\begin{aligned} P(A_i | E) &= \frac{P(A_i \cap E)}{P(E)} \\ &= \frac{P(A_i)P(E | A_i)}{P(A_1)P(E | A_1) + P(A_2)P(E | A_2) + \dots + P(A_k)P(E | A_k)} \end{aligned}$$

Examples [\[edit \]](#)

Drug testing [\[edit \]](#)

Suppose, a particular test for whether someone has been using cannabis is 90% **sensitive**, meaning the **true positive rate** (TPR)=0.90. Therefore it leads to 90% true positive results (correct identification of drug use) for cannabis users.

The test is also 80% **specific**, meaning **true negative rate** (TNR)=0.80. Therefore the test correctly identifies 80% of non-use for non-users, but also generates 20% false positives, or **false positive rate** (FPR)=0.20, for non-users.

Assuming 0.05 **prevalence**, meaning 5% of people use cannabis, what is the **probability** that a random person who tests positive is really a cannabis user?

The **Positive predictive value** (PPV) of a test is the proportion of persons who are actually positive out of all those testing positive, and can be calculated from a sample as:

$$\text{PPV} = \text{True positive} / \text{Tested positive}$$

If sensitivity, specificity, and prevalence are known, PPV can be calculated using Bayes theorem. Let $P(\text{User} \mid \text{Positive})$ mean "the probability that someone is a cannabis user given that they test positive," which is what is meant by PPV. We can write:

$$\begin{aligned} P(\text{User} \mid \text{Positive}) &= \frac{P(\text{Positive} \mid \text{User})P(\text{User})}{P(\text{Positive})} \\ &= \frac{P(\text{Positive} \mid \text{User})P(\text{User})}{P(\text{Positive} \mid \text{User})P(\text{User}) + P(\text{Positive} \mid \text{Non-user})P(\text{Non-user})} \\ &= \frac{0.90 \times 0.05}{0.90 \times 0.05 + 0.20 \times 0.95} = \frac{0.045}{0.045 + 0.19} \approx 19\% \end{aligned}$$

The fact that $P(\text{Positive}) = P(\text{Positive} \mid \text{User})P(\text{User}) + P(\text{Positive} \mid \text{Non-user})P(\text{Non-user})$ is a direct application of the **Law of Total Probability**. In this case, it says that the probability that someone tests positive is the probability that a user tests positive, times the probability of being a user, plus the probability that a non-user tests positive, times the probability of being a non-user.

This is true because the classifications user and non-user form a **partition of a set**, namely the set of people who take the drug test. This combined with the definition of **conditional probability** results in the above statement.

Even if someone tests positive, the probability they are a cannabis user is only 19%, because in this group only 5% of people are users, most positives are false positives coming from the remaining 95%.

If 1,000 people were tested:

- 950 are non-users and 190 of them give false positive (0.20×950)
- 50 of them are users and 45 of them give true positive (0.90×50)

The 1,000 people thus yields 235 positive tests, of which only 45 are genuine drug users, about 19%. See Figure 1 for an illustration using a frequency box, and note how small the pink area of true positives is compared to the blue area of false positives.

Sensitivity or specificity [\[edit \]](#)

The importance of **specificity** can be seen by showing that even if sensitivity is raised to 100% and specificity remains at 80%, the probability of someone testing positive really being a cannabis user only rises from 19% to 21%, but if the sensitivity is held at 90% and the specificity is increased to 95%, the probability rises to 49%.

Test \ Actual	User	Non-user	Total
Positive	45	190	235
Negative	5	760	765
Total	50	950	1000

90% sensitive, 80% specific, $\text{PPV} = 45/235 \approx 19\%$

Test \ Actual	User	Non-user	Total
Positive	50	190	240
Negative	0	760	760
Total	50	950	1000

100% sensitive, 80% specific, $\text{PPV} = 50/240 \approx 21\%$

Test \ Actual	User	Non-user	Total
Positive	45	47	92
Negative	5	903	908
Total	50	950	1000

90% sensitive, 95% specific, $\text{PPV} = 45/92 \approx 49\%$

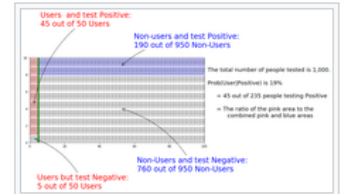
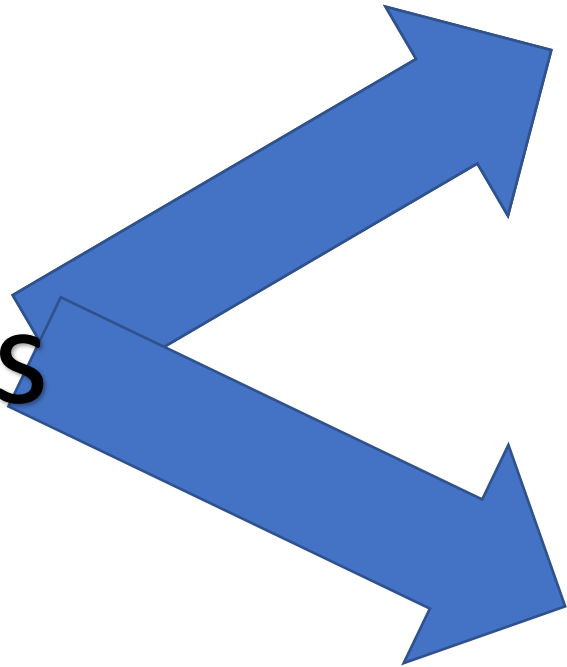


Figure 1: Using a frequency box to show $P(\text{User} \mid \text{Positive})$ visually by comparison of areas

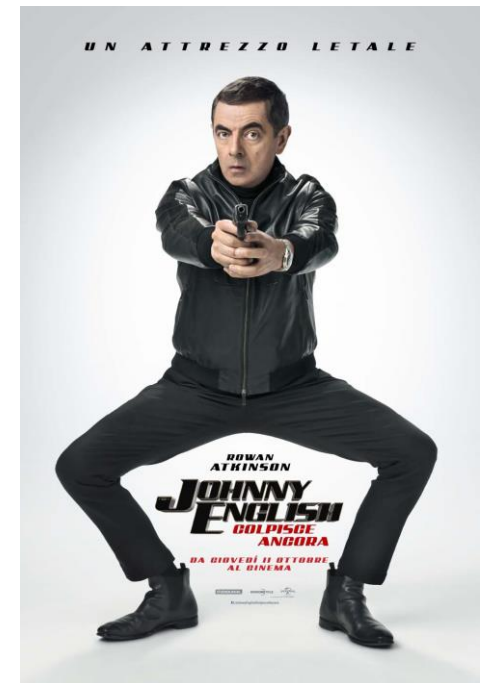
Bayes' rule – 2 usages

2 Ways



**Invert
probability cond.
 $P(B|A) \rightarrow P(A|B)$**

**A whole new way to
do statistics**



This year
we will do
some
Bayesian
analysis



Bayesian Simple Linear Regression

```
data {  
  int<lower=0> N;  
  vector[N] x;  
  vector[N] y;  
}  
parameters {  
  real beta0;  
  real beta1;  
  real<lower=0> sigma;  
}  
model {  
  y ~ normal(beta0 + beta1 * x, sigma);  
  beta0 ~ normal(0,100);  
  beta1 ~ normal(0,100);  
  sigma ~ gamma(1,1);  
}
```

Counting Rules

Example 3.20

Multiplicative Rule:
Routing Problem

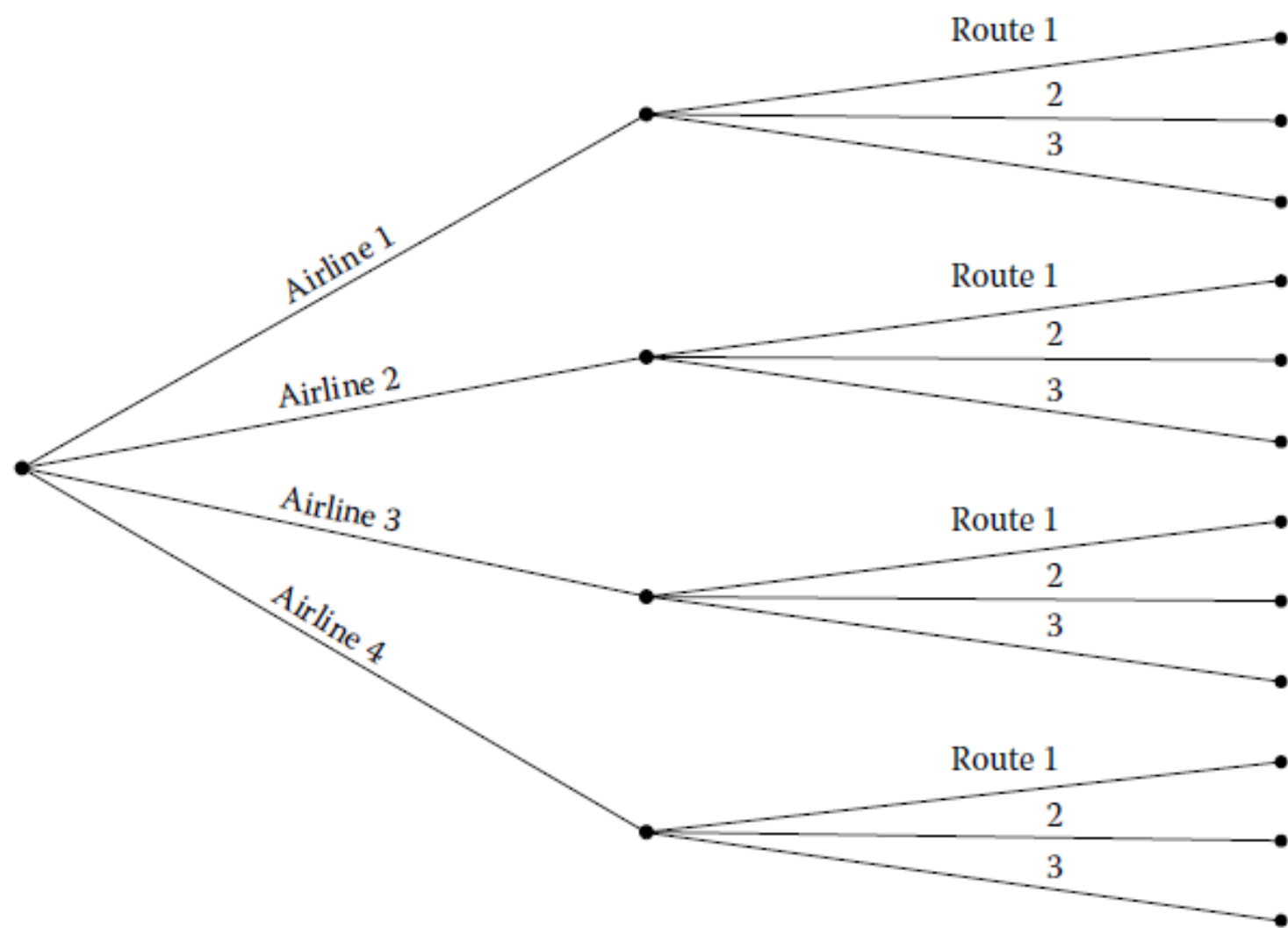
Solution

A product (e.g., hardware for a networked computer system) can be shipped by four different airlines, and each airline can ship via three different routes. How many distinct ways exist to ship the product?

A pictorial representation of the different ways to ship the product will aid in counting them. This representation, called a **decision tree**, is shown in Figure 3.13. At the starting point (stage 1), there are four choices—the different airlines—to begin the journey. Once we have chosen an airline (stage 2), there are three choices—the different routes—to complete the shipment and reach the final destination. Thus, the decision tree clearly shows that there are $(4)(3) = 12$ distinct ways to ship the product.

FIGURE 3.13

Decision tree for shipping problem



THEOREM 3.1

The Multiplicative Rule You have k sets of elements— n_1 in the first set, n_2 in the second set, . . . , and n_k in the k th set. Suppose you want to form a sample of k elements by taking one element from each of the k sets. The number of different samples that can be formed is the product

$$n_1 n_2 n_3 \cdots n_k$$

Example 3.21

Multiplicative Rule:
Candidate Selection Problem

Solution

There are 20 candidates for three different mechanical engineer positions, E_1 , E_2 , and E_3 . How many different ways could you fill the positions?

This example consists of the following $k = 3$ sets of elements:

Set 1: Candidates available to fill position E_1

Set 2: Candidates remaining (after filling E_1) that are available to fill E_2

Set 3: Candidates remaining (after filling E_1 and E_2) that are available to fill E_3

The numbers of elements in the sets are $n_1 = 20$, $n_2 = 19$, $n_3 = 18$. Therefore, the number of different ways of filling the three positions is

$$n_1 n_2 n_3 = (20)(19)(18) = 6,840$$

THEOREM 3.2

Permutations Rule Given a single set of N distinctly different elements, you wish to select n elements from the N and arrange them within n positions in a distinct order. The number of different permutations of the N elements taken n at a time is denoted by P_n^N and is equal to

$$P_n^N = N(N - 1)(N - 2) \cdots (N - n + 1) = \frac{N!}{(N - n)!}$$

where $n! = n(n - 1)(n - 2) \cdots (3)(2)(1)$ and is called n factorial. (Thus, for example, $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$.) The quantity $0!$ is defined to be equal to 1.

THEOREM 3.3

Partitions Rule There exists a single set of N distinctly different elements and you want to partition them into k sets, the first set containing n_1 elements, the second containing n_2 elements, . . . , and the k th set containing n_k elements. The number of different partitions is

$$\frac{N!}{n_1!n_2!\cdots n_k!} \quad \text{where } n_1 + n_2 + n_3 + \cdots + n_k = N$$

Example 3.26

Partitions Rule: Another
Assignment Problem

Solution

You have 12 system analysts and you want to assign three to job 1, four to job 2, and five to job 3. In how many different ways can you make this assignment?

For this example, $k = 3$ (corresponding to the $k = 3$ different jobs), $N = 12$, $n_1 = 3$, $n_2 = 4$, and $n_3 = 5$. Then the number of different ways to assign the system analysts to the jobs is

$$\frac{N!}{n_1!n_2!n_3!} = \frac{12!}{3!4!5!} = \frac{12 \cdot 11 \cdot 10 \cdot \cdots \cdot 3 \cdot 2 \cdot 1}{(3 \cdot 2 \cdot 1)(4 \cdot 3 \cdot 2 \cdot 1)(5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)} = 27,720$$

THEOREM 3.4

The Combinations Rule A sample of n elements is to be chosen from a set of N elements. Then the number of different samples of n elements that can be selected from

N is denoted by $\binom{N}{n}$ and is equal to

$$\binom{N}{n} = \frac{N!}{n!(N - n)!}$$

Note that the order in which the n elements are drawn is not important.

Summary of Counting Rules

1. *Multiplicative rule:* If you are drawing one element from each of k sets of elements, with the sizes of the sets n_1, n_2, \dots, n_k , the number of different results is

$$n_1 n_2 n_3 \cdots n_k$$

2. *Permutations rule:* If you are drawing n elements from a set of N elements and arranging the n elements in a distinct order, the number of different results is

$$P_n^N = \frac{N!}{(N - n)!}$$

3. *Partitions rule:* If you are partitioning the elements of a set of N elements into k groups consisting of n_1, n_2, \dots, n_k elements ($n_1 + n_2 + \dots + n_k = N$), the number of different results is

$$\frac{N!}{n_1! n_2! \cdots n_k!}$$

4. *Combinations rule:* If you are drawing n elements from a set of N elements without regard to the order of the n elements, the number of different results is

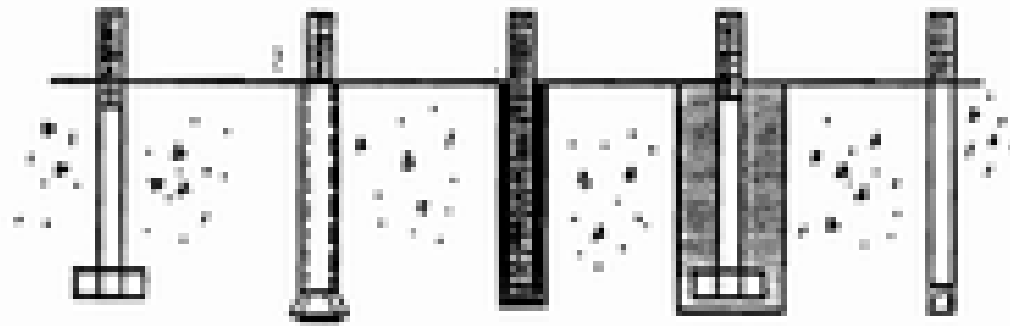
$$\binom{N}{n} = \frac{N!}{n!(N - n)!}$$

(Note: The combinations rule is a special case of the partitions rule when $k = 2$.)

Birthday Problem (Use $P(A) = 1 - P(A^c)$)

- What is the probability that two or more people in a random sample of size k people have a common birthday?
- Let A be the event 2 or more people of the k have the same birthday
- A^c is the event all people of the k have different birthdays

```
> birthday <- function(k){  
+ 1 - exp(lchoose(365,k) + lfactorial(k) - k*log(365))  
+ }  
  
> birthday(20:24)  
  
[1] 0.4114384 0.4436883 0.4756953 0.5072972 0.5383443
```



Cast-in-Place Undercut Adhesive Grouted Expansion

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Studs

Suppose a firm that manufactures concrete studs is researching the hypothesis that its new chemically anchored studs achieve greater holding capacity and greater carrying load capacity than the more conventional, mechanically anchored studs. To test the hypothesis, three new chemical anchors are selected from a day's production and subjected to a durability test. Each of the three $\frac{1}{2}$ -inch studs is drilled and set into a slab of 4,000 pounds-per-square-inch stone aggregate concrete, and their tensile load capacities (in pounds) are recorded. It is known from many previous durability tests of mechanically anchored studs that approximately 16% of mechanical anchors will have tensile strengths over 12,000 pounds. Suppose that all three of the chemically anchored studs tested have tensile strengths greater than 12,000 pounds. What can researchers for the firm conclude?

The connection between probability and statistics

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To answer these questions, define the events

A_1 : {Chemically anchored stud 1 has tensile strength over 12,000 pounds}

A_2 : {Chemically anchored stud 2 has tensile strength over 12,000 pounds}

A_3 : {Chemically anchored stud 3 has tensile strength over 12,000 pounds}

We want to find $P(A_1 \cap A_2 \cap A_3)$, the probability that all three tested studs have tensile load capacities over 12,000 pounds.

Since the studs are selected by chance from a large production, it may be plausible to assume that the events A_1 , A_2 , and A_3 are independent. That is,

$$P(A_2|A_1) = P(A_2)$$

In words, knowing that the first stud has a tensile strength over 12,000 pounds does not affect the probability that the second stud has a tensile strength over 12,000 pounds. With the assumption of independence, we can calculate the probability of the intersection by multiplying the individual probabilities:

$$P(A_1 \cap A_2 \cap A_3) = P(A_1)P(A_2)P(A_3)$$

If the new chemically anchored studs are no stronger or no weaker than the mechanically anchored studs, that is, *if the relative frequency distribution of tensile strengths for chemically anchored studs is no different from that for mechanically anchored studs*, then we would expect about 16% of the new studs to have tensile strengths over 12,000 pounds. Consequently, our estimate of $P(A)$ is .16 for all three studs, and

$$P(A_1 \cap A_2 \cap A_3) \approx (.16)(.16)(.16) = .004096$$

Thus, the probability that the firm's researchers will observe all three studs with tensile load capacity over 12,000 pounds is only about .004. If this event were to occur, the researchers might conclude that it lends credence to the theory that chemically anchored studs achieve greater carrying load capacity than mechanically anchored studs, *since it is so unlikely to occur if the distributions of tensile strength are the same*. Such a conclusion would be an application of the rare event approach to statistical inference. You can see that the basic principles of probability play an important role.